Maclaurin Series:

(interval of convergence centered on

$x=0$.

So interval of convergence is:

- $\{0\}$ .... only converges for $x=0$
- $(-a,a)$ .... for some positive real # $a$

  [[we call the radius of convergence $a$]]
- $(-\infty,\infty)$.... all real #s

  [[radius of convergence is $\infty$]]

Problems/Question about interval of convergence

↳ interval of X-values

$\frac{1}{1-x} = 1 + x + x^2 + \cdots$

$\xi$

→ what about the endpts of that interval?

$(-a, a)$

what about $x = -a$?

and $x = a$?

Ans ⇒ check each one separately

Note that...

when we say "converge" on the open interval...

it is absolute convergence

Convergence at the endpts may be absolute or conditional.

To determine radius of convergence:

RATIO TEST

Taylor series

what if I want to know about a point NOT near $x=0$?

Ans: just translate the polynomial...

## Maclaurin Series

$$f(x) = \sum_{k=0}^{\infty} \frac{f^{(k)}(0)}{k!} x^k$$

## Taylor Series (centered around $x_0$)

$$f(x) = \sum_{k=0}^{\infty} \frac{f^{(k)}(x_0)}{k!} (x-x_0)^k$$

10.8/25-45 odd

25) $\sum_{k=0}^{\infty} \frac{x^k}{k+1}$

Radius of convergence [ratio test]

$$\lim_{k\to\infty}\left|\frac{a_{k+1}}{a_k}\right| = \lim_{k\to\infty}\left|\frac{\left(\frac{x^{k+1}}{(k+1)+1}\right)}{\left(\frac{x^k}{k+1}\right)}\right| =$$

$$\lim_{k\to\infty}\left|\frac{x^{k+1}}{k+2}\cdot\frac{k+1}{x^k}\right| = \lim_{k\to\infty}\left|x\cdot\frac{k+1}{k+2}\right|$$

$$= |x| \lim_{k\to\infty}\left|\frac{k+1}{k+2}\right| = |x|$$

Ratio Test: if limit < 1 series converges

$|x| < 1$

so the radius of convergence is 1

the preliminary int. of conv is $-1 < x < 1$

when $x = -1$, series is $\sum_{k=0}^{\infty} \frac{(-1)^k}{k+1}$

but this is alt. harmonic ... ∴ conv

when $x = 1$, series is $\sum_{k=0}^{\infty} \frac{1^k}{k+1}$

the harmonic series, ... ∴ Diverges

So int of convergence is

$[-1, 1)$

Mar 29-8:52 AM